JOURNAL OF SPACECRAFT AND ROCKETS
Vol. 39, No. 6, November–December 2002

Optimal Spacecraft Rendezvous Using Genetic Algorithms

Young Ha Kim* and David B. Spencer†
Pennsylvania State University, University Park, Pennsylvania 16802

The optimal rendezvous of two spacecraft are examined using a genetic algorithm. The minimum fuel solution of the optimal rendezvous contains many local optima, as well as discontinuities in the solution. These local optima and discontinuities make locating a global optimal solution difficult. Genetic algorithms are effective in solving these kinds of problems. The goal is to find the thrust time history that includes the magnitude and direction of the velocity change and the burn position, such that the boundary conditions are satisfied to an acceptable level and in a reasonable time. In addition, the number of thrust arcs and the maximum magnitude of the velocity change are regulated. This method was used on three test cases: 1) the Hohmann transfer, 2) the bielliptic transfer, and 3) rendezvous with two impulses. The results of the Hohmann and the bielliptic transfers match the analytical solutions within the resolution of the variables of the genetic algorithm. Although the result from the rendezvous with two impulses is not exact, the configuration of the trajectory is similar to the analytical solution.

Nomenclature

C	=	weighting factor (associated with position and velocity vectors)
$f(M, n)$	=	fitness of each individual population
L	=	length of the chromosome
M	=	generation number
$M_{\max}$	=	maximum number of generations
$N_{\max}$	=	maximum number of members in a generation
n	=	individual member of a generation
$P(M)$	=	population at Mth generation
P_c	=	crossover probability
P_m	=	mutation probability
R	=	radius ratio for bielliptic transfer
r	=	magnitude of position vector, km
$\boldsymbol{r}(t)$	=	position vector at time t, km
$\boldsymbol{r}(t_f)$	=	position vector after travel time t_f (refers to chaser or target), km
t_f	=	travel time, s
v	=	magnitude of velocity vector, km/s
$\boldsymbol{v}(t)$	=	velocity vector at time t, km/s
$\boldsymbol{v}(t_f)$	=	velocity vector after travel time t_f (refers to chaser or target), km/s
x, y	=	orbit plane vector components, km
Δv	=	magnitude of the velocity change, km/s
$\Delta \boldsymbol{v}(t)$	=	velocity change generated by the genetic algorithm at time t, km/s
$\Delta \boldsymbol{v}_{\min}$	=	velocity change known by the analytical solution of Hohmann transfer, km/s
θ	=	true anomaly at the time of a maneuver, deg
ϕ	=	the direction of $\Delta \boldsymbol{v}$, deg

Subscripts

c	=	chaser vehicle
cx, cy	=	chaser x and y components
i, f	=	initial, final
t	=	target vehicle
tx, ty	=	target x and y components
1, 2, 3	=	order of occurrence

Introduction

THE total solution to the optimal spacecraft rendezvous problem contains many local optima with discontinuous parts between them that can inhibit convergence to the global optimal solution. Conventional calculus-based optimization methods are not effective in these kinds of problems because the optima they seek are the best in the neighborhood of the current point and are dependent on the existence of derivatives. These conventional methods require an accurate initial guess to identify promising trajectories; unfortunately, it is not always easy to determine the initial guess. The goal of this paper is to introduce the use of genetic algorithms for optimal space rendezvous. At the time a rendezvous sequence is initiated, the two space vehicles may be far apart in significantly different orbits. The rendezvous is accomplished when both space vehicles attain the same position vector and velocity vector at the same time. In this paper, only coplanar transfers are analyzed because there are well-known analytical solutions (Hohmann and bielliptical transfers) with which to compare results. The initial and final orbits define the boundary conditions such that the initial orbit is the chase vehicle's orbit and the final orbit is the target vehicle's orbit.

Optimal Rendezvous Using a Genetic Algorithm

The goal of the optimal rendezvous is to obtain a thrust time history, which includes the thrust direction, magnitude, and the burn time, such that the boundary conditions are satisfied to an acceptable level, and to provide these solutions in a reasonable time. The transfer of a spacecraft from one point in space to another is a fundamental problem in astrodynamics known as Lambert's problem. Rendezvous is Lambert's problem where the space vehicle matches both the position and velocity of the target. For the general orbit transfers, three variables per trajectory segment need to be coded into the genetic algorithm. In the case of a rendezvous problem, the variables are Δv, θ, and ϕ. These variables are shown in Fig. 1. A space vehicle moving with initial velocity $\boldsymbol{v}(t)$ leaves the initial orbit $\boldsymbol{r}(0)$ after thrusting. The velocity of the space vehicle becomes $\boldsymbol{v}'$, which is the summation vector of the initial velocity vector $\boldsymbol{v}(0)$ and the velocity change vector $\Delta \boldsymbol{v}$. The space vehicle arrives in the final orbit if the vehicle has the suitable thrust profile. The genetic algorithm finds the optimal thrust profile, similar to what was done in Refs. 1 and 2 for orbit transfers. Where this work differs is that a second maneuver is made causing a rendezvous to occur.

Genetic Algorithms

The genetic algorithm (GA) is a stochastic global search method that mimics some aspects of natural biological evolution. The GA operates on a population of potential solutions by applying the principle of survival of the fittest to converge to an optimal solution. At

Received 30 January 2002; revision received 3 September 2002; accepted for publication 3 September 2002.

*Graduate Student, Department of Aerospace Engineering; currently Graduate Student, Department of Aerospace Engineering, Texas A&M University, College Station, TX 77843. Student Member AIAA.

†Assistant Professor, Department of Aerospace Engineering; dbs9@psu.edu. Associate Fellow AIAA.

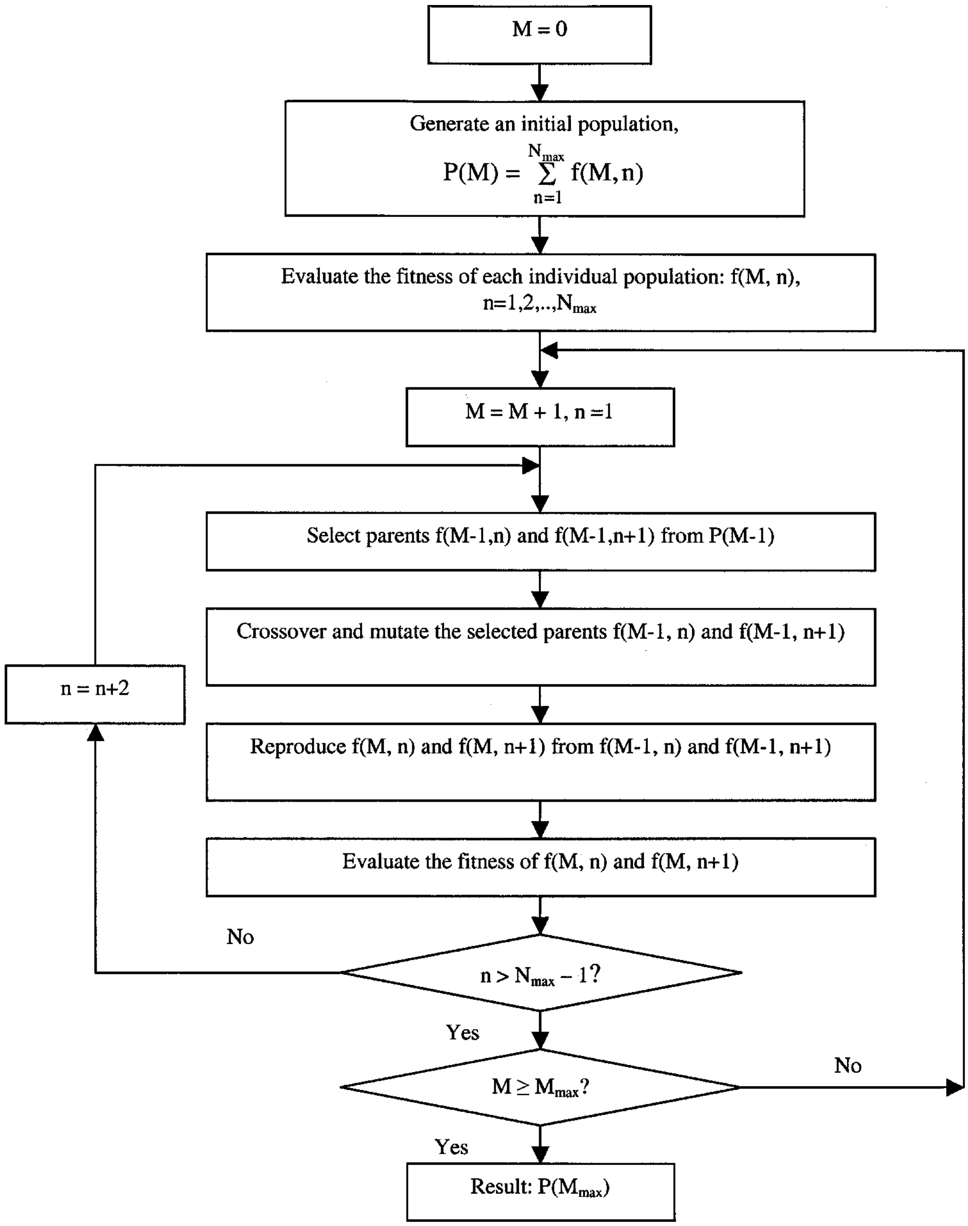

Fig. 1 Geometry of an orbit transfer.

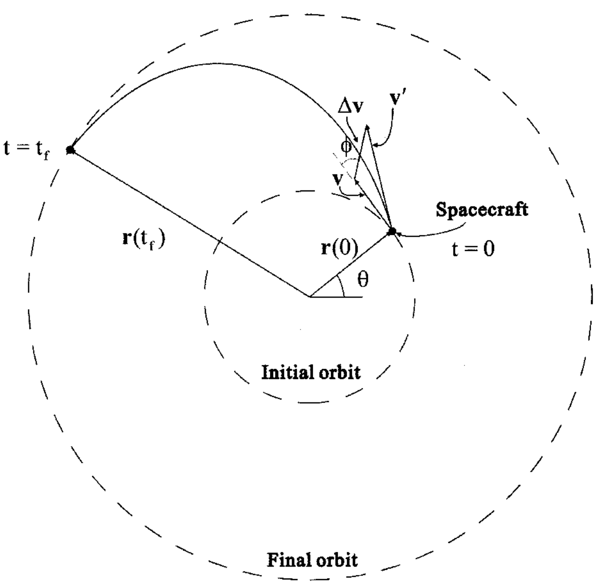

Fig. 2 Outline of a GA.

each generation, where a generation is one iteration of the GA, a new set of approximations is created by the processes of selecting individuals according to their level of fitness in the problem domain and breeding them together using operators borrowed from natural genetics. This natural selection leads to the evolution of populations of individuals that are better suited to their environment than the individuals that they were created from. More information on GAs may be found in Ref. 3, among others.

To use a GA, the problem solution is represented as a genome (or chromosome). The GA then creates a population of solutions and applies genetic operators such as reproduction, crossover, and mutation to evolve the solutions to find the fittest one(s). Based on the these principles, the simple GA used is illustrated in the flowchart in Fig. 2. More details on the GA developed for this study may be found by Kim.[4]

Major Elements of the GA

This algorithm introduces many concepts, including the fitness of a chromosome, the selection probability of a chromosome for parenthood of the next generation, the crossover operator to exchange a bit string, and the mutation operator to introduce random perturbations in the search. A brief summary of the components used in this study is now presented.

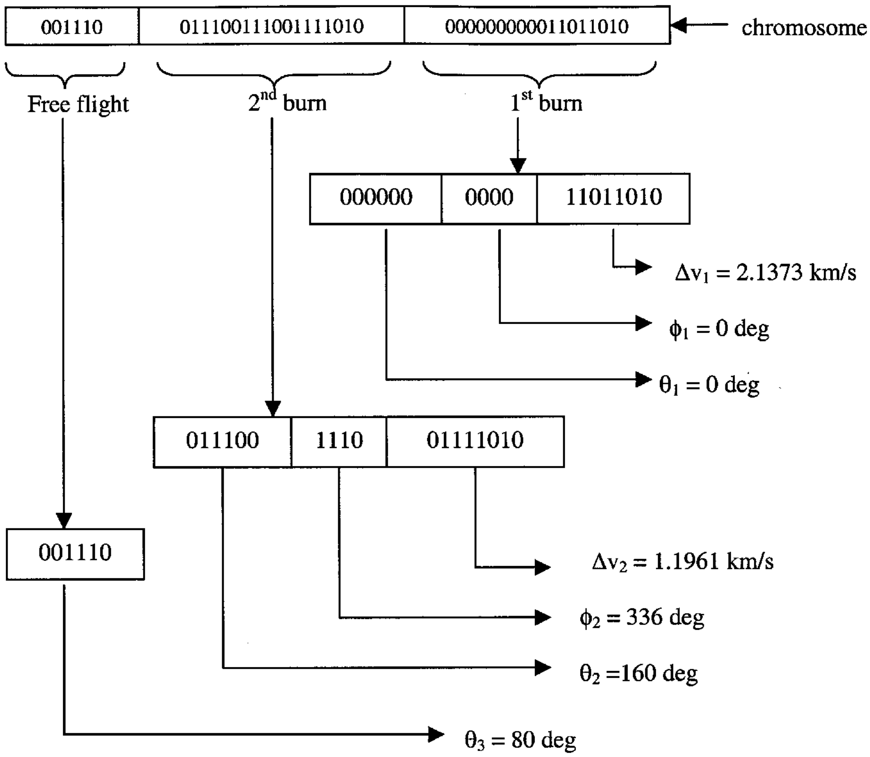

Fig. 3 Structure of chromosome.

Encoding and Decoding of the Variables

Each variable in the GA is encoded in binary form. The number of bits for each variable is determined by the required accuracy. The example in Fig. 3 is the case of a two-impulse transfer, and the chromosome consists in two impulse flights and one free flight without thrust. The magnitude of velocity change $\Delta \boldsymbol{v}(t)$ is encoded with 8 bit, the burn position (true anomaly θ) is 6 bit, and the direction of $\Delta \boldsymbol{v}(t)$, ϕ, is 4 bit. The structure of an example individual chromosome is indicated in Fig. 3. Details about the coding in this chromosome are presented by Kim.[4]

Figure 4 shows the trajectory decoded from the chromosome in Fig. 3. The spacecraft thrusts in the initial orbit. When it reaches the position that the true anomaly increases 160 deg from initial position in the transfer orbit, the second burn occurs. The spacecraft coasts for another 80 deg in true anomaly after the second burn. The trajectory in Fig. 4 is not optimized; it is just an example.

Fitness Function

To evaluate the fitness of the individual, a fitness function must be defined. The fitness function is designed to evaluate the final position and velocity of the spacecraft at the final time. The positions and velocities at the final time t_f of the target spacecraft and chase spacecraft are assumed to be satisfied if the boundary conditions are within some tolerance (chosen here as 1%) of the actual solution for the final orbit. This tolerance is included because it is very difficult to make these conditions exactly meet the theoretical boundary conditions. Equation (1) is a general form of the fitness function:

$$\text{fitness} = 1/[1 + C_r\|\boldsymbol{r}_t(t_f) - \boldsymbol{r}_c(t_f)\| + C_v\|\boldsymbol{v}_t(t_f) - \boldsymbol{v}_c(t_f)\|] \quad (1)$$

The weighting factors C_r and C_v are chosen based on which variables of the fitness function are more important. For example, if the accuracy of the final position is more important than the accuracy of the final velocity, C_r has a large value relative to C_v. The magnitudes of the variables also influence the weighting factor. The maximum value of the fitness function is 1 and is chosen in this form so that the problem is one of maximizing the fitness.

A coplanar Hohmann transfer from low Earth orbit (LEO) to geosynchronous Earth orbit (GEO) was modeled in this paper because the optimal solution is known and to compare the results is easy. The Hohmann transfer consists of two impulses with the thrust time history divided into two segments. For Hohmann transfer from LEO to GEO, the open time transfer fitness function appears in Eq. (2) as

$$\text{fitness} = 1\Big/\Big[1 + 10^{-4} \times \|x_t - x_c(t_f)\| + 10^{-4} \times \|y_t - y_c(t_f)\| + 10^{-1} \times \|v_{tx} - v_{cx}(t_f)\| + 10^{-1} \times \|v_{ty} - v_{cy}(t_f)\|\Big] \quad (2)$$

In Eq. (2), if the position $[x_c(t_f), y_c(t_f)]$ and velocity $[v_{cx}(t_f), v_{cy}(t_f)]$ of the chase spacecraft is equal to the position $[x_t, y_t]$ and velocity $[v_{tx}, v_{ty}]$ of the target spacecraft at the final time, the fitness will be unity, which is the maximum fitness value. The weighting factors used in Eq. (2) are determined by trial and error and were chosen after examining about 10 different combinations.

Table 1 Selected values of the GA parameters

Parameter	Selected value
Population size	150
Generation	100
Probability of crossover	0.8
Probability of mutation	0.002

Selection of the GA Parameters

The simple GA by Goldberg[3] is used in this analysis. Several improved ideas were added, such as elitism[5] and the tournament selection method.[5] Four parameters in a GA are used to find the solution: the size of population, the number of generations, the probability of crossover, and the probability of mutation. Selecting an optimal combination of four GA parameters is very difficult because each of the GA parameters is varied individually and the number of combinations of the four parameters is infinite. Although the values of Table 1 are not necessarily optimal, these values are used in three applications this paper, and are based partly on recommended practices presented by De Jong.[6]

Whole Algorithm of Optimal Rendezvous Using a GA

The entire algorithm of optimal rendezvous using a GA consists of the dynamics segment and the GA segment. The concept of the

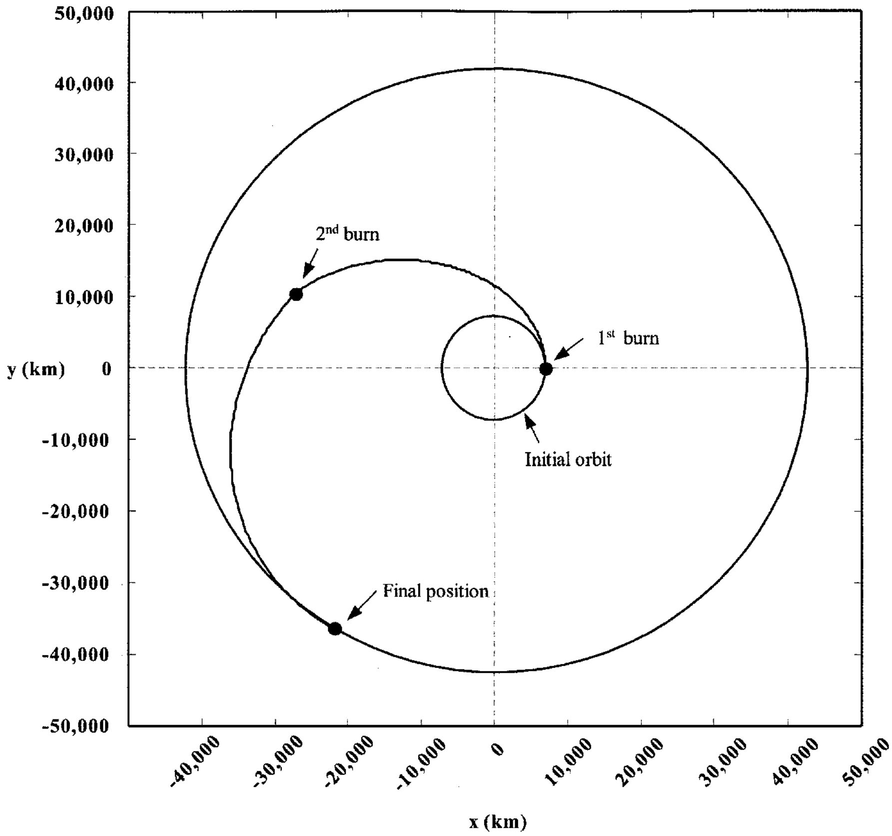

Fig. 4 Decoded trajectory from the chromosome in Fig. 3.

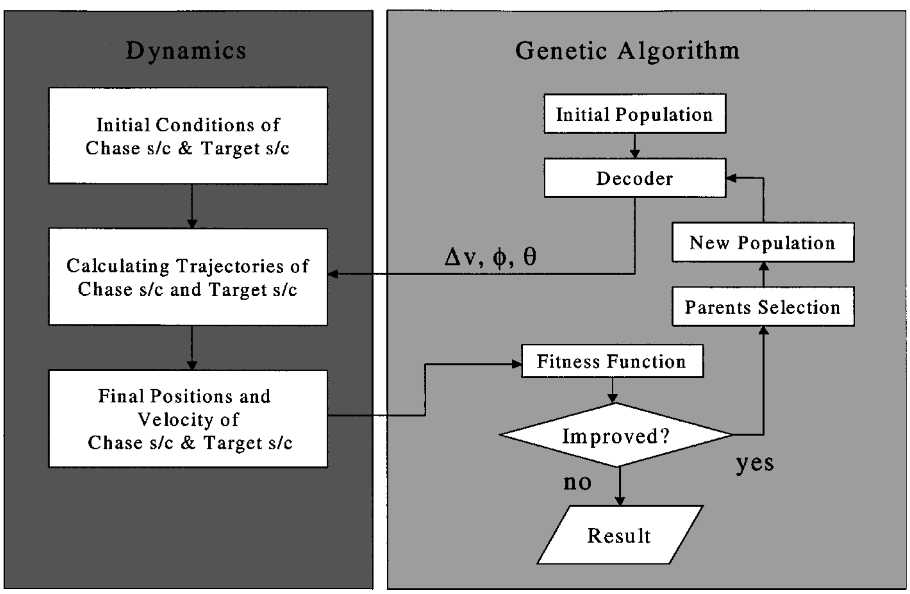

Fig. 5 Whole algorithm of optimal rendezvous using a GA.

whole algorithm follows the flowchart in Fig. 5. The trajectories of the two space vehicles are determined by the initial conditions and the thrust time history generated by the GA segment. The fitness of the trajectories evaluated by the fitness function is used as the criterion for selecting parents. The selected parents are used to generate the new generation that is decoded into the thruster time history. The procedure is repeated until the defined nth generation is reached or no improvement from generation to generation is achieved.

Results

The GA is applied to three types of transfers. The first is the Hohmann transfer, the second is the bielliptic transfer, and the last is a rendezvous with two impulses. Because the solutions of the first and second transfer are known, these transfers are used to compare with the solution of the GA. In the rendezvous with two impulses, the GA is applied to find an unknown solution.

Hohmann Transfer

The Hohmann transfer is the minimum propellant two-impulse transfer between coplanar circular orbits. Refer to Fig. 6; the Hohmann transfer consists of two relatively simple maneuvers. Here, the GA is applied to a Hohmann transfer from LEO ($r = 7000$ km) to GEO ($r = 42{,}164$ km). The Hohmann transfer consists of two impulses with the thrust time history divided into two segments. The genetic parameters in Table 1 are used to apply the GA. The fitness function is given by

$$\text{fitness} = 1/\left[1 + 10^{-6} \times \|r_t - r_c(t_f)\| + 10^{-2} \times \|v_t - v_c(t_f)\| + 1 \times \|\Sigma \Delta V_{\min} - \Sigma \Delta V\|\right] \tag{3}$$

This augmented fitness function is used because the optimal solution is already known. This function penalizes solutions that are further away from the optimal solution. The weighting factors in Eq. (3) are decided, as before, by trial and error.

The solution generated by the GA is given in Table 2, and Fig. 7 shows the trajectory and the thrust profile of this solution. The fitness of the solution is 0.9945. The trajectory in Fig. 8 has a similar configuration as the trajectory in Fig. 6. The difference between the Figs. 8 and 6 occurs due to the resolution of the variables in the encoded chromosome.

Whereas the optimal values of θ_1 and θ_2 are 180 deg each, the values of θ in Table 2 are 182.9 and 177.1 deg. These are the closest values to 180 deg that can be generated by the GA used, because the resolution of the bits in the 6-bit chromosome for θ is 5.8 deg [$360\text{ deg}/(2^5 + 2^4 + 2^3 + 2^2 + 2^1 + 2^0) = 5.8$ deg].

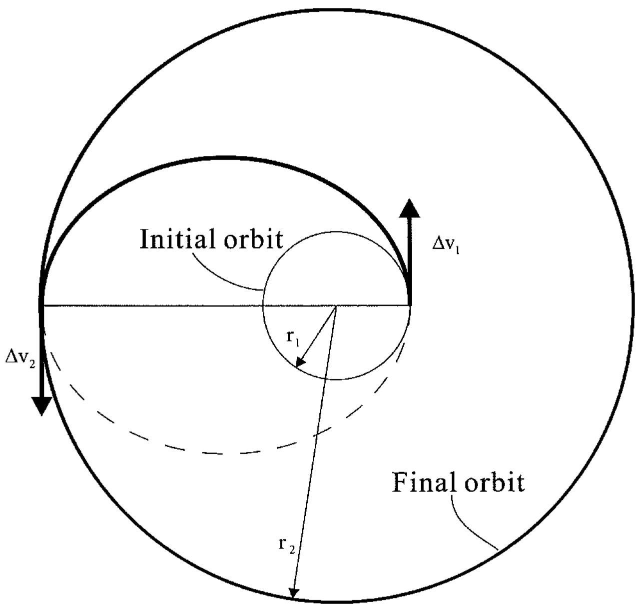

Fig. 6 Geometry of the Hohmann transfer.

Bielliptic Transfer

The bielliptic transfer is a three-impulse transfer between coplanar circular orbits. The geometry of Fig. 7 shows that the transfer begins with a Δv_1 applied tangentially to the circular orbit velocity. This Δv_1 is larger than the first impulse of a corresponding Hohmann transfer because the apogee radius r_2 of the resulting transfer ellipse is larger than the final circular orbit radius r_f. At apogee in this first transfer ellipse, Δv_2 is applied tangentially to the existing apogee velocity. The magnitude of Δv_2 is determined by the requirements to raise the perigee radius of the resulting transfer ellipse from r_1 to r_f. At perigee in this second transfer ellipse, Δv_f is applied tangentially but opposite to the direction of motion. The magnitude of Δv_f is the difference between the perigee velocity of the second transfer ellipse and the final circular orbit velocity. For radius ratio $(R = r_f/r_1) > 15.58$, any bielliptic transfer where the midcourse impulse lies outside the outer orbit is more economical than the Hohmann transfer. For $11.94 < R < 15.58$, the bielliptic transfer is more economical than the Hohmann transfer only if the midcourse impulse location is sufficiently large.[7] Here, the GA is applied to a bielliptic transfer from LEO ($r_1 = 7000$ km) to GEO ($r_f = 42{,}164$ km). Although the minimum-fuel transfer is the Hohmann transfer in this problem, the bielliptic transfer is tried to apply a three-impulse transfer. The values of genetic parameters in Table 1 are used in this problem. The fitness function used is the same equation, Eq. (3).

The solution generated by the GA is given in Table 3. Figure 9 shows the trajectory and the thrust profile of solution by the GA. The fitness of the solution is 0.9251. This value is lower than the fitness

Table 2 Hohmann transfer solution generated by the GA

Maneuver	GA Δv, km/s	Hohmann Δv, km/s	ϕ, deg	GA θ, deg	Hohmann θ, deg
First burn	2.343	2.336	0	0	0
Second burn	1.431	1.434	0	182.9	180
Free flight	0	0	0	177.1	180

Table 3 Bielliptic transfer solution generated by the GA

Burn	Δv, km/s	ϕ, deg	θ, deg
First	2.839	0	0
Second	0.706	0	182.9
Third	1.004	182.9	177.1

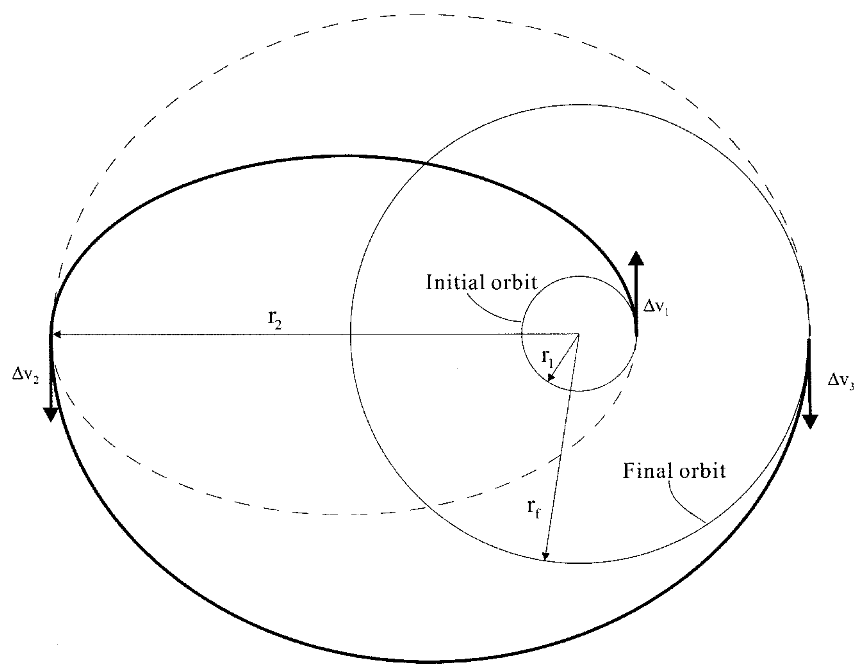

Fig. 7 Geometry of the bielliptic transfer.

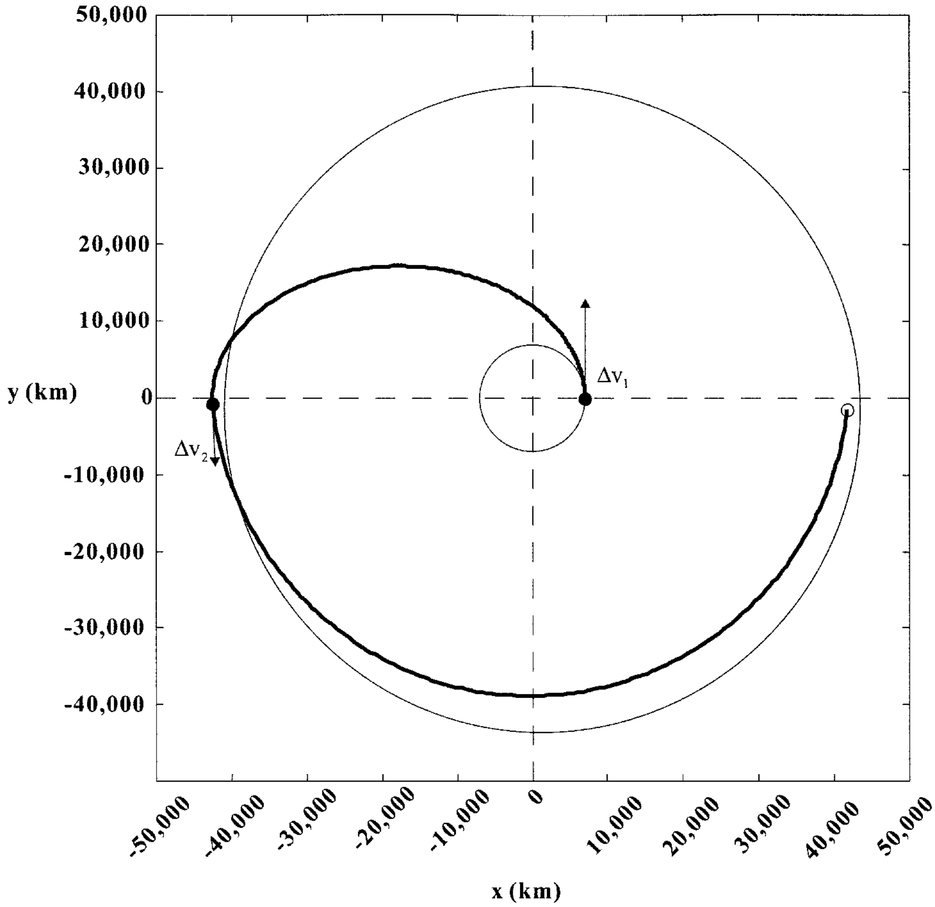

Fig. 8 Trajectory of Hohmann transfer generated by the GA.

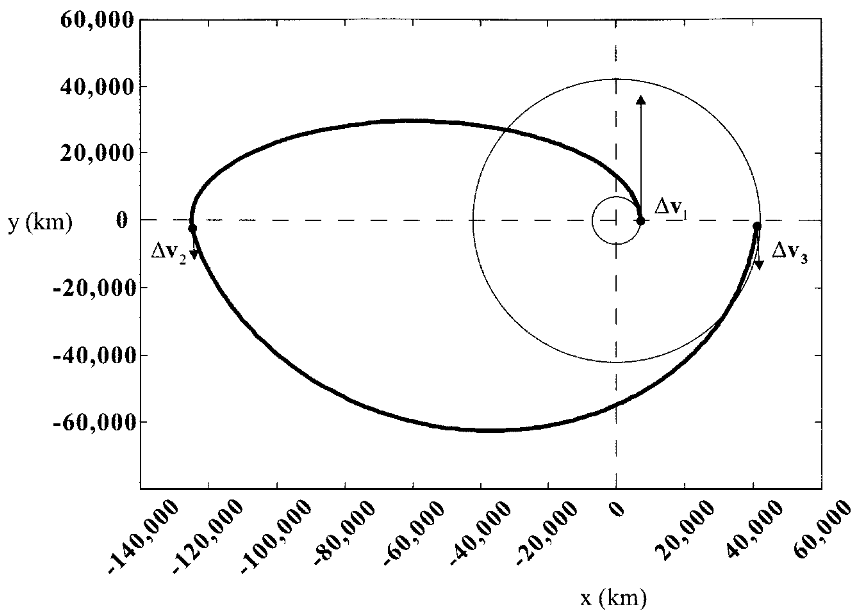

Fig. 9 Trajectory of the bielliptic transfer generated by the GA.

of the solution in Hohmann transfer. Nevertheless, the trajectory in Fig. 9 has the typical appearance of the bielliptic transfer.

Rendezvous with Two Impulses

In the Hohmann and the bielliptic transfers, the goal of the chase vehicle is to reach a determined position on the final orbit. In the problem of rendezvous with two impulses, the target is moving on the final orbit, where the value of the true anomaly at the intercept point is not 180 deg ahead of the initial point (like the Hohmann transfer). The chase and target vehicles start at periapsis. This problem is more complicated than the preceding two problems, although the same fitness function and the same GA parameters are used again.

The solution generated by the GA is given in Table 4. Figure 10 shows the trajectory and the thrust profile of solution by the GA. The fitness of the solution is 0.8548. This value is lower than the fitness of solutions in the Hohmann transfer and in the bielliptic transfer. However, the configuration of the trajectory is similar to the trajectory of the Hohmann transfer.

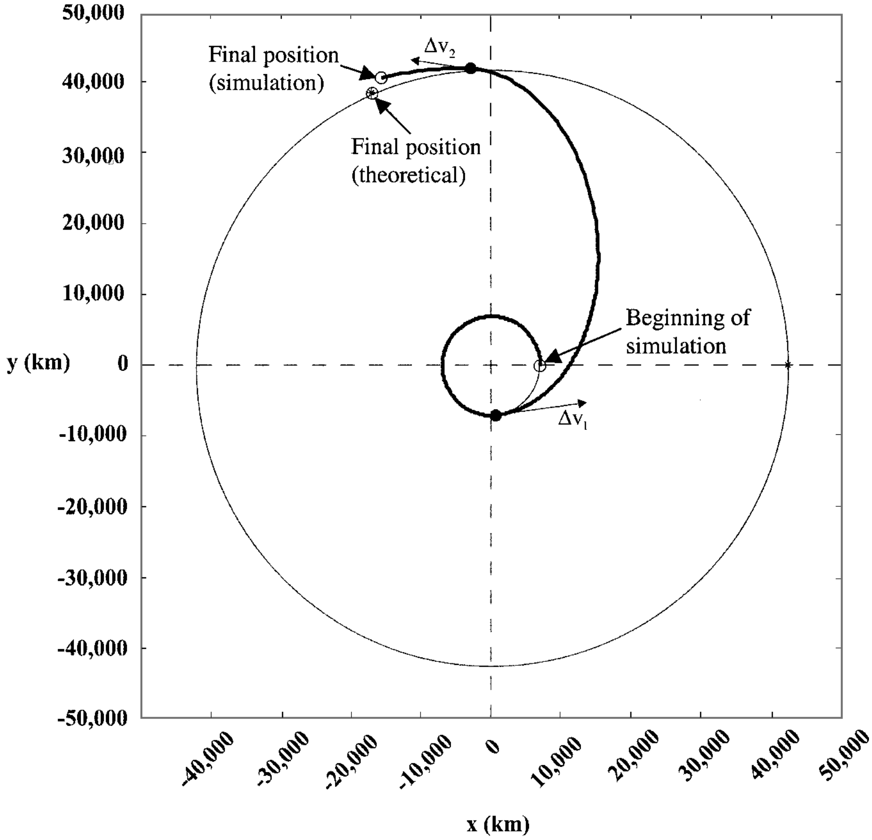

Fig. 10 Trajectory of the two-impulse rendezvous generated by the GA.

Table 4 Two-impulse rendezvous solution generated by the GA

Maneuver	Δv, km/s	ϕ, deg	θ, deg
First burn	2.343	360	277.8
Second burn	1.575	360	177.1
Free flight	0	0	17.0

Conclusions

The goal of this paper was to introduce the use of GAs for optimal rendezvous. Because the GA is a simple and useful method in the optimization, it can be used in various problems just by changing the fitness function without modifying the other parts of the algorithm. However, it is not always true that the GA is always advantageous. When the GA is applied to some problems, many genetic parameters and variable have to be determined. Each problem requires some combination of the genetic parameters. It is not always a simple matter to determine the parameters because it is difficult to understand how each value affects the problem. Increasing or decreasing the number of bits in the chromosome, for example, can change the resolution of the problem variables (Δv and angles). However, this comes at the expense of increased or decreased computational cost.

Besides the genetic parameters, it is also difficult to find the weighting factors of the fitness function. In this paper, those values were found by trial and error. Various methods to find these values have been introduced in many other fields. If the genetic parameters and weighting factors are found more easily, the GA is more useful and effective.

The fitness function used in this paper also has room for improvement. It was made by intuition; other fitness functions can be used and could result in differing values in fitness. Even with the same encoded chromosome, different fitness functions will have different fitness values. To determine the best fitness function is not simple because it is not developed using a rigorous mathematical proof. Various trial and error tests are needed to improve the fitness function. However, general methods to improve the function exist. For example, if constraints are added in any fitness function, some disqualified solutions are penalized, and relatively suitable solutions result in better fitness values.

Even with the limitations we imposed in this study, we were able to successfully demonstrate that a GA can find a close approximation to known spacecraft rendezvous solutions. This gave us the confidence to apply these methods to examples where an analytical solution was not known.

References

[1]Rauwolf, G. A., and Coverstone-Carroll, V. L., "Near-Optimal Low-Thrust Orbit Transfers Generated by a Genetic Algorithm," *Journal of Spacecraft and Rockets*, Vol. 33, No. 6, 1996, pp. 859–862.

[2]Rauwolf, G. A., "Near-Optimal Low-Thrust Orbit Transfers Generated by a Genetic Algorithm," M.S. Thesis, Aeronautical and Astronautical Engineering Dept., Univ. of Illinois, Urbana, IL, Aug. 1995.

[3]Goldberg, D. E., *Genetic Algorithms in Search, Optimization, and Machine Learning*, 1st ed., Addison Wesley Longman, Reading, MA, 1989, pp. 7–14, 115.

[4]Kim, Y. H., "Optimal Rendezvous Using a Genetic Algorithm," M.S. Thesis, Dept. of Aerospace Engineering, Pennsylvania State Univ., University Park, PA, Aug. 2001.

[5]Haupt, R. L., and Haupt, S. E., *Practical Genetic Algorithms*, Wiley-Interscience, New York, 1998, pp. 40, 41.

[6]De Jong, K. A., "Analysis of the Behavior of a Class of Genetic Adaptive Systems," Ph.D. Dissertation, Dept. of Computer Science, Univ. of Michigan, Ann Arbor, MI, May 1975.

[7]Prussing, J. E., and Conway, B. A., *Orbital Mechanics*, Oxford Univ. Press, New York, 1993, pp. 108–113.

C. A. Kluever
Associate Editor